Optimal generation investment under suboptimal dispatch

A bilevel equilibrium model of optimal investment incentives in zonal markets

Anselm Eicke, Hertie School, eicke@hertie-school.org

When choosing where to place a new generator, a tradeoff arises between the cost of power generation and the network costs: to which extend do lower generation costs merit higher network expenditures? In most power systems, however, private investors rarely consider the effect of their decisions on the network because they do not pay the associated costs. A solution to this problem is the internalization of network costs via locational signals that provide (dis-) incentives to invest in certain regions. This study estimates how such locational investment incentives affect welfare in a zonal electricity market with suboptimal dispatch incentives. For this aim, a novel game theoretical model is proposed that reflects the Stackelberg relationship between a regulator (leaders) and private generation companies (followers). In the first stage, the regulator first chooses a locational investment signal with the aim of maximizing social welfare. In the second stage, generators decide on investment and dispatch based on a uniform electricity price and the location-specific investment signal. Applying the model to an exemplary test network reveals that locational signals may significantly reduce the need for network expansion through a better spatial distribution of generators. I also find that the optimal siting of generators in a zonal market is closer to consumers than it would be in a nodal market, in which both dispatch and investment incentives are optimal. This is due to the additional redispatch costs which arise from the suboptimal dispatch incentives of a zonal market.

Keywords: Stackelberg game, Locational signals, Zonal electricity markets, Investment incentives

Highlights

- The optimal placement of generators differs between zonal and nodal markets
- Compared to nodal markets, generators and consumers are closer to each other in zonal markets because the suboptimal dispatch results in additional redispatch costs
- The case-study reveals welfare improvements through an optimal placement of generators even if the subsequent dispatch is suboptimal
- Optimal locational investment signals differ between locations and technologies, which is not the case in most real-world instruments
- The novel modelling approach allows the representation of intra-zonal network constraints while maintaining a uniform electricity price

1. Introduction

Placement of generators. The decarbonization of the power sector has profound implications on the spatial distribution of generators. Because the potential of renewable energy sources, such as wind and solar, differs strongly between locations, the construction of renewable energy sources is concentrated in some regions, which are often far from consumption centers. Transmission infrastructure must be extended to cope with these new spatial generation patterns, which is costly and takes years, if not decades, to build.

Tradeoff between network and generation cost. A crucial question for the siting of new generators is hence to which extend lower generation costs merit higher network expenditures. Without accounting for this tradeoff between generation costs and network costs, cost-benefit analyses are likely to be misleading (Borenstein 2012). However, most European countries have structured their national electricity market as a single price zone and socialize the network costs. Such a setting does not reflect electricity flows and line congestions and therefore fails to provide the price signals that inform investors about the cost for network reinforcement and investment. In economic terms, the costs resulting from the siting decisions are externalized. Absent of regulatory intervention, investors face no economic incentive to consider the effect of their siting decisions on infrastructure requirements in zonal markets. As consequence, the resulting strain on the transmission system becomes increasingly problematic in many European countries. Two prominent examples are Germany and the UK, where wind generation is concentrated in the northern part while load centers are in the south of the countries. Costa-Campi (2020) highlight that most generators in Spain, above all wind power, are inefficiently located regarding consumption, which implies a significant overinvestment in the transmission network.

Internalization of network costs. The standard economic solution to address such a trade-off is the internalization of externalities. If investors face all costs resulting from their investment decision it is rational for them to account for network and generation cost.

Locational markets. Locational pricing is seen by many as the solution to this problem (e.g. Neuhoff et al. 2013). Locational electricity markets lead to prices that differ by location. The best-known example of locational markets are locational marginal prices (LMPs), where prices are determined at each node in the transmission system.[1] Another form of locational markets is market splitting: a large uniform pricing zone is divided into multiple smaller zones. In theory, locational markets are clearly the best approach to provide adequate dispatch and investment incentives for generators: a nodal market is therefore also used as a reference scenario later in the analysis. In practice, however, locational markets may not lead to optimal investment decisions. An econometric analysis of Texas's electricity market by Brown et al. (2020) suggests that other factors than nodal prices drive location decisions for utility-scale generation investments. Not only are these price signals non-decisive, they also only reflect the short-run and not the long-run network costs in practice. Perez-Arriaga et al. (1995) find that nodal prices only recover about 30% of the network costs due to lumpiness in network investment. This implies that the locational investment signals arising from nodal prices are far below efficient long-term incentives.

Regulatory locational signals. Another approach to internalize network costs is the introduction of regulatory locational signals. In zonal and in locational power markets, regulators around the world introduced locational price signals to (dis-) incentivize investment in certain regions (Eicke et al. 2020). In practice, such locational signals are implemented as grid connection or grid usage charges,

[1] Also known as nodal prices

are built into capacity mechanisms or part of renewable support schemes, e.g., as location specific subsidy. Eicke et al. (2020) reveal that none of these approaches prevails in practice: Locational investment signals are diverse in design, and there is no common methodology to determine their magnitude.

Literature on locational investment signals. Literature on the effect of locational signals in zonal markets is scarce. A few papers evaluate locational signals that are determined based on hypothetical nodal prices. For instance, Schmidt and Zinke (2020) estimate locational signals as the difference between nodal and zonal market values, which allows them to base their analysis on numerically well suited zonal and nodal power market model. Grimm et al. (2019) use signals that reflect average nodal electricity prices and are therefore uniform for all technologies. Locational signals can also apply for consumers of electricity. Von Scheidt et al. (2021) analyze the effect of siting electrolyzers based on nodal prices in a zonal system. While all these approaches are likely to lead to improved investment decisions of generators and consumers, they ignore that the dispatch under zonal pricing differs from the nodal dispatch. Consequently, the proposed locational signals and resulting distribution of generators are not optimal.

Research question. The core objective of this paper is to shed light on the effect of optimally designed locational investment signals in zonal electricity markets. More explicitly, the paper addresses the following two research questions:

1. What is the welfare-optimal distribution of generators given the suboptimal dispatch incentives of a zonal market?
2. How must location-specific investment signals be designed to lead to this distribution?

Methodological approach. The methodology of the paper is a bilevel model, which is implemented as a Mathematical Problem with Equilibrium Constraints (MPEC). This model setup captures the Stackelberg game between a regulator, who determines the cost-minimizing level of network charges, and agents on a competitive electricity market. It determines the welfare-optimal distribution of generators in a zonal power system and the locational price signals that lead to this distribution. The aim of this paper is to estimate the optimal level of such a locational instrument and to analyze its effect; the practical implementation of the locational signal is not discussed. If deemed helpful, the locational signal can be interpreted as an annualized grid connection charge on generators, but it is not specific to this form of implementation.

Findings. The main contribution of this paper is a better understanding of the effects of locational investment signals. For this aim, I present a novel modelling approach that determines welfare-optimal locational investment signals. Applying this approach on an exemplary power system reveals fundamental properties of optimal locational instruments and their impact on the distribution of power generation. I find that optimal investment incentives may lead to significant welfare improvements. Further, I show that the optimal distribution of generation depends on the market design: the optimal siting in a zonal power system differs from the optimal distribution in a nodal power system due to differences in the dispatch of generation.

Outline of the paper. In what follows, I present the employed model (Section 2) and the exemplary power system used for the case-study (Section 3). The model results are presented, and their relevance is discussed in Section 4. Section 5 summarizes the main findings and concludes.

2. Model: approach and mathematical formulation

In the following section, I present the model applied in this paper (Subsection 2.1) and introduce its notation (Subsection 2.2) as well as its mathematical formulation. Subsection 2.3 concludes with a solution strategy to overcome the computational challenges of the presented modelling approach.

2.1. Bilevel model of a single bidding zone with locational instruments

Locational instruments in practice. In practice, most locational instruments identify the responsibility of each generator on the network costs. These costs are allocated on generators (and sometimes consumers) based on cost-responsibility – like a Pigouvian tax. A list of practical examples of locational instruments is provided by Eicke et al. (2020). The precise methodology for such cost-internalization is, however, ambiguous and differs strongly among regulators. The implementation of such an approach in a long-term model is challenging because it requires an iterative calculation of investment decisions to reach an equilibrium state.

One-shot model. In this contribution, I propose an alternative methodology. Instead of explicitly internalizing the network costs, I determine the level of locational signals that maximizes social welfare. This approach has the strong advantage that the regional distribution of generators resulting from welfare optimizing network charges can be assessed in a one-shot model. Absent of market failures, both approaches lead to the same results because the internalization of all externalities is necessary and sufficient for a market to lead to the social optimum.

Basic setup. Prior to determining the welfare-optimal locational signal, I briefly describe how locational signals affect social welfare (**Error! Reference source not found.**). I assume that the locational investment signal is defined by the regulatory authority. The signal imposes additional investment costs on generators and thereby affects investment decisions of generators, who are driven by profit maximization. This also alters dispatch and consumption decisions and thereby affects the flow on transmission lines. Based on these requirements, the transmission system operator (TSO) decides on network investment and redispatch measures in to minimize the overall network costs. The welfare level results from the consumer surplus, generation costs and network costs. When the regulator can anticipate the behavior of the market participants and the TSO, it can optimize the signals such that it maximizes welfare. Two aspects are worth noting here. First, I

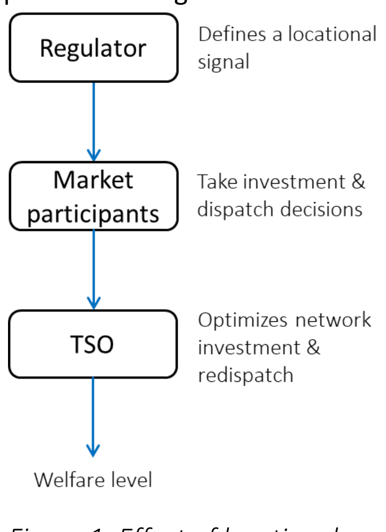

Figure 1: Effect of locational investment signals on social welfare

analyze the long-term equilibrium resulting from locational investment signals. Therefore, I assume that the regulator can anticipate the behavior of rational and non-strategic market participants. Second, the objective functions of TSO and regulator are aligned because both aim for maximizing the social welfare. In addition, the decisions of the TSO do not affect the decisions of the market participants due to the zonal market design. Hence, the regulator and the TSO can be modeled as a single agent which acts first.

Methodological approach. I apply a game theory to determine the welfare-optimal locational investment signals in a zonal power market. I interpret the interaction between the regulator and participants of the electricity market as a strategic game. In this game, the regulator first chooses the level of a locational signal that maximizes social welfare. In a second step, generators decide on the investment and the dispatch of power generation, while accounting for the locational instrument and the price signals in a zonal power market. The spot market is assumed to be perfectly competitive, i.e., no firm has the potential to affect prices through strategic behavior. Because the regulator (leader) anticipates the response of the participants in the wholesale electricity market (follower), I model these two interlinked optimization problems as a Stackelberg game (Figure 2). A feature of this approach is that the regulator accounts for the network topology, while the participants in the zonal electricity market do not. This stands in contrast with most other system models that are either zonal or nodal on all levels.

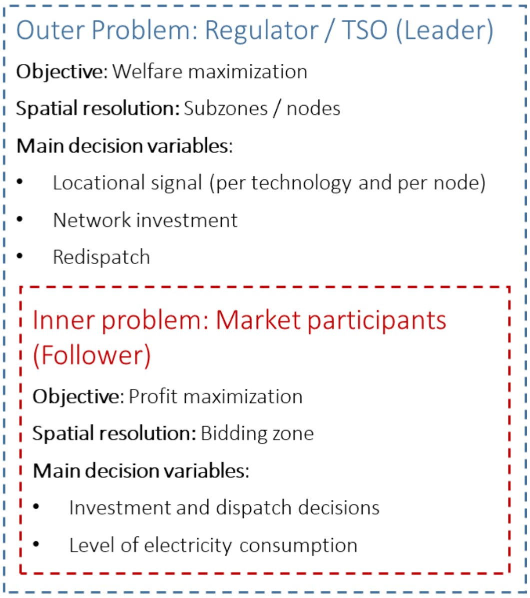

Figure 2: The interaction between regulator, TSO and market participants as a Stackelberg game

2.2. Technical and economic setup

Next, I present the notation and economic setup that is used throughout the paper.

2.2.1. Time horizon and network model

I consider a set of equidistantly discretized timesteps $T = \{t_1 \dots t_{|T|}\}$, an electricity transmission network with a set of nodes $N = \{n_1 \dots n_{|N|}\}$, and the set of transmission lines between nodes $L = \{l_{n_1-n_2} \dots l_{n_n-n_m}\}$. All lines are characterized by their capacity $\bar{f}$, which limits the maximal power flow on each line. Physical flow restrictions are modelled according to the lossless DC approach introduced by Schweppe et al. (1988), which is a linearization of Kirchhoff's laws.

2.2.2. *Electricity generation: investment and dispatch*

Investment and dispatch. The presented model is a greenfield model which optimizes the dispatch of and investment in power generation. The set of candidate generation technologies is called $Z = \{z_1 \dots z_{|Z|}\}$. These technologies differ by their investment costs $c_{z,n}^{fix}$ and their variable costs $c_{z,n}^{var}$. Some generators are limited by an availability factor $\alpha_{t,z,n}$, which varies over time and location. It can be interpreted as the wind or solar potential or as power plant outages.

Rising capacity costs. To account for a reduced profitability of sites at increasing deployment of each technology at a specific node, I introduce rising marginal investment costs at each location: The capacity-cost of a technology at a specific location is composed of the fixed investment costs and a linear price adder m_s. Absent a locational signal, this price adder ensures an equal distribution of generation capacity between nodes. These increasing marginal capacity costs are also necessary to apply locational steering through price signals. Without increasing marginal costs, a locational signal in one area would lead to a relocation of all capacity.

Generation costs. I summarize the cost of investment in generation assets and the cost of their dispatch under the term *generation costs* (GC):

$$GC = \sum_{z,n} P_{z,n} \cdot \left(c_{z,n}^{fix} + P_{z,n} \cdot m_s\right) - \sum_{t,z,n}\left(G_{t,z,n} \cdot c_{z,n}^{en}\right) \quad (1)$$

2.2.3. *Network investment and redispatch*

Network investment. Power flows in the transmission network are limited by the line capacity. The optimal line capacity is determined endogenously in the model and depends above all on the investment costs for each transmission line l, which are denoted as c_l^{grid}.

Redispatch measures. To avoid very low utilization rates of the peak line capacity, the cost-optimal transmission network is sometimes congested. Instead of expanding the transmission network for the highest power flows, redispatch measures are activated to address line congestions. In the model, cost-based redispatch is assumed, as it is currently applied in the Austrian, Swiss, and German markets. In case of congestion, some generators before the bottleneck are downward dispatched ($R_{t,z,n}^{down}$) and compensated for their foregone revenues.[2] The compensation is calculated as the difference between wholesale price λ_t and variable costs of each generator $c_{z,n}^{var}$. It is necessary to cover the generator's contribution margin and implies an additional cost for the operation of the power system. Behind the bottleneck, the same amount of generation that was previously not dispatched is upward dispatched $R_{t,z,n}^{up}$ and compensated for its variable cost. The cost-based compensation makes generators indifferent towards redispatch and eliminates incentives for strategic behavior. To ensure sufficient upwards capacity for redispatch measures, investment in reserve capacity $P_{z,n}^{reserve}$ is also allowed in the model.

Network costs. I summarize all transmission related costs under the term *network costs* (NC), which cover transmission lines investments and the cost of redispatch:

$$NC = \sum_l \overline{f_l} \cdot c_l^{grid} + \sum_{t,z,n}\left[R_{t,z,n}^{up} \cdot c_{z,n}^{var} - R_{t,z,n}^{down} \cdot \left(\lambda_t - c_{z,n}^{var}\right)\right] \quad (2)$$

[2] Previous paper that applying a similar bilevel model formulation neglect the compensation of foregone profits (Grimm et al. 2016; 2019). While this strongly simplifies the mathematical solution of the model, it does not render participants of the wholesale markets indifferent about the redispatch. In that case, anticipatable redispatch measures affect siting decisions.

2.2.4. Electricity demand and consumer surplus

Demand function. I assume a linear, price-elastic demand function with a negative slope. Electricity demand $d_{t,n}$ therefore depends on the endogenous zonal electricity price λ_t, which reflects the marginal cost of power generation, and on an exogenous reference demand $d_{t,n}^{ref}$ and price p_t^{ref}. The following inverse demand function has the short-term price elasticity ε and is linear (Annex B):

$$\lambda_t(d_{t,n}) = p_t^{ref} \cdot \left[1 - \frac{1}{\varepsilon} + \frac{d_{t,n}}{\varepsilon \cdot d_{t,n}^{ref}}\right] \tag{3}$$

Consumer surplus. The gross consumer surplus (GCS) describes the value all consumers obtain from the consumption of electricity. It is pictured by the area below the inverse demand function and can hance be derived as the integral of the inverse demand function between zero and the realized electricity consumption.

$$GCS \; = \; \sum_{t,n} \int_0^{d_{t,n}} \lambda_t(u)\, du = \; \sum_{t,n} p_t^{ref} \cdot d_{t,n} \cdot \left[1 - \frac{1}{\varepsilon} + \frac{d_{t,n}}{2\varepsilon \cdot d_{t,n}^{ref}}\right] \tag{4}$$

2.2.5. Social welfare function

Social welfare. To compare different scenarios, I assume a social welfare function. I define the social welfare (SW) as the difference between the gross consumer surplus (42.2.5) and the cost of electricity supply, which comprises generation (1) and network costs (2):

$$SW = GCS - GC - NC \tag{5}$$

2.2.6. Locational signals

Locational signals. Locational signals arise from differences in generation costs or availability between nodes. In addition, the regulator can impose additional locational signals on generators, so called G-components, through location-specific charges or bonuses. I analyze locational signals that are specified as capacity-based charges, i.e., a fee charged for each MW connected to the network. A capacity-based charge is independent of the dispatch and therefore does not affect dispatch decision of generators. A practical example are the British "Transmission Network Use of System" fees that are collected based on power plant capacity connected to the national grid.

Table 1 summarizes the nomenclature of all variables and parameters. Note that the distinction between variables and parameters is ambiguous because some variables of the outer problem are parameters in the inner problem.

Table 1: Nomenclature

Sets	
t	Hour
z	Technology
n	Subzone / node within the power system
Decision variables and derived quantities	
$G_{t,z,n}$	Generation of technology z in hour t and at node n
$P_{z,n}$	Installed capacity of technology z at node n
$d_{t,n}$	Actual load (accounting for price elasticity)
$f_{l,t}$	Power flow from node n to node m

$\overline{f_l}$	Installed capacity of transmission line l
$R^{up}_{t,z,n}$	Upward redispatch
$R^{down}_{t,z,n}$	Downward redispatch
$S_{z,n}$	Locational signal for technology z at node n (parameter in the inner problem)
λ_t	Dual variable of the zonal energy balance: the market clearing price
Parameters	
$\alpha_{t,z,n}$	Availability of technology tec in hour t and node n
$d^{ref}_{t,n}$	Time series of the reference level of electricity demand (prior to price elasticity)
$c^{var}_{z,n}$	Variable cost of electricity generation for tec at node n
$c^{fix}_{z,n}$	Fixed costs of electricity generation for tec at node n
$c^{instr}_{z,n}$	Energy- or capacity-based location- and technology-specific network charge
c^{grid}_{l}	Network investment costs
$\overline{P}_{z,n}$	Upper limit for the installable capacity of tec at node n
m_s	Increase of the cost of supply at higher penetration rates

2.3. Mathematical formulation as a bilevel model

Game theoretical approach. The problem exhibits a bilevel structure. The regulator first determines the level of locational signal that maximizes social welfare (outer problem). Based on these signals, generators decide in a second step on the investment and the dispatch of power generation (inner problem). Both levels are presented in the following.

2.3.1. Outer problem: Determination of locational signal, network investment and operation

Objective function. The aim of the regulator is to maximize the social welfare (Equation 6). However, investment and dispatch decisions are taken by the market participants. To affect their decisions, the regulator can introduce a locational signal $S_{z,n}$.

$$\max_{S_{z,n},\overline{f_l},f_{l,t},R^{up}_{t,z,n},R^{down}_{t,z,n},G_{t,z,n},P_{z,n},d_{t,n}} \sum_{t,n} p^{ref}_t \cdot d_{t,n} \cdot \left[1 - \frac{1}{\varepsilon} + \frac{d_{t,n}}{2\varepsilon \cdot d^{ref}_{t,n}}\right] - P_{z,n} \cdot \sum_{z,n} \left(c^{fix}_{z,n} + P_{z,n} \cdot m_s\right)$$
$$- \sum\nolimits_{t,z,n} \left(G_{t,z,n} \cdot c^{en}_{z,n}\right) - \sum\nolimits_l \overline{f_l} \cdot c^{grid}_l - \sum\nolimits_{t,z,n} \left[R^{up}_{t,z,n} \cdot c^{var}_{z,n} + R^{down}_{t,z,n} \cdot \left(\lambda_t - c^{var}_{z,n}\right)\right] \quad (6)$$

Constraints. Next to the inner problem that is presented in Subsection 2.3.2, several constraints limit the feasible space of the outer problem.

Nodal energy balance. *Kirchhoff's first law* constitutes the conservation of power flows and determines the flows in and out of each node, which corresponds to an energy balance of each node. Because the zonal electricity market from the inner problem does not necessarily lead to a feasible dispatch, upward and downward redispatch measures may be needed. Combined, this leads to the nodal energy balance

$$\sum\nolimits_z \left[G_{t,z,n} + R^{up}_{t,z,n} - R^{down}_{t,z,n}\right] - d_{t,n} = \sum\nolimits_{l\ out} f_{l,t} + \sum\nolimits_{l\ in} f_{l,t} \qquad \forall\ t, l, n \qquad (7)$$

Power flow constraints. *Kirchhoff's second law* describes the distribution of flows across transmission lines depending on the voltage angles at each node. It is modeled according to the DC approach for lossless transmission, which is a linear approximation by means of the voltage phase angle θ at each node.

$$f_{i\rightarrow j,t} = B \cdot \left(\theta_{i,t} - \theta_{j,t}\right) \qquad \forall\ t, (i,j) \in N \qquad (8)$$

$$f_{i\rightarrow j,t} \leq \overline{f_{i\rightarrow j}} \qquad \forall \text{ t}, (\text{i}, \text{j}) \in \text{N} \qquad (9)$$

To obtain unique physical solutions, the voltage phase angle θ is fixed at an arbitrary node $n^* \in$ N:

$$\theta_{n^*,t} = 0 \qquad \forall \text{ t} \qquad (10)$$

Redispatch constraints. Further constraints are introduced to specify the redispatch measures. Downward redispatch (or curtailment) is limited by generation – only previously dispatched generation can be lowered.

$$R_{t,z,n}^{down} \leq G_{t,z,n} \qquad \forall \text{ t, z, n} \qquad (11)$$

Upward redispatch is limited by the available capacity that does not already generate.

$$R_{t,z,n}^{up} \leq P_{z,n} \cdot \alpha_{t,z,n} - G_{t,z,n} \qquad \forall \text{ t, z, n} \qquad (12)$$

Summary. To sum up, the outer problem is the following inter-temporal minimization problem:

Maximize social welfare

s.t. Nodal energy balance (6)

Power flow restrictions (7-10)

Redispatch constraints (11-12)

2.3.2. *Inner problem: Profit maximization of agents in the wholesale electricity market*

Profit maximization. The inner problem represents investment in generation capacity and spot market trading by private firms. It can be represented as the profit maximization of generators and consumers (Equation 14). I assume a perfectly competitive market, i.e., all agents are price-takers. The network costs do not appear in this profit maximization, because they are not internalized and hence do not affect the profit of market participants. By contrast, the locational instrument $S_{z,n}$ affects the profits of private firms.

$$\max_{G_{t,z,n},P_{z,n},d_{t,n}} \sum_{t,z,n} G_{t,z,n} \cdot (\lambda_t - c_{z,n}^{var}) - \sum_{z,n} \left[P_{z,n} \cdot \left(c_{z,n}^{fix} + P_{z,n} \cdot m_c + S_{z,n}\right)\right]$$
$$+ \sum_{t,n} p_t^{ref} \cdot d_{t,n} \cdot \left[1 - \frac{1}{\varepsilon} + \frac{d_{t,n}}{2\varepsilon \cdot d_{t,n}^{ref}}\right] - d_{t,n} \cdot \lambda_t \qquad (14)$$

Constraints. Several constraints limit the feasible space of this inner problem. They are presented in the following. The dual variables of each constraint are annotated right to each equation in Greek letters.

Zonal energy balance. First, all electricity demand must be served, which corresponds to a zonal energy balance (Equation 15). The dual variable of the zonal energy balance corresponds to the electricity price, which is uniform within the entire pricing zone. Equation 15 allows to further simplify Equation 14: the terms containing the electricity price cancel out.

$$\sum_{\text{n}} d_{t,n} - \sum_{\text{z,n}} G_{t,z,n} = 0 \qquad : \lambda_t \qquad \forall \text{ t} \qquad (15)$$

Production constraints. Further, generation of each generator is limited by the installed capacity and the availability; generation, capacity and consumption cannot become negative, and an upper limit of the installable capacity is introduced (Equations 16-20).

$G_{t,z,n} - \alpha_{t,z,n} \cdot P_{z,n} \leq 0$	$: \overline{\mu^G_{t,z,n}}$	$\forall$ t, z, n	(16)
$- G_{t,z,n} \leq 0$	$: \underline{\mu^G_{t,z,n}}$	$\forall$ t, z, n	(17)
$- P_{z,n} \leq 0$	$: \underline{\mu^C_{z,n}}$	$\forall$ z, n	(18)
$- d_{t,n} \leq 0$	$: \underline{\mu^D_{t,n}}$	$\forall$ t, n	(19)
$P_{z,n} - \bar{P}_{z,n} \leq 0$	$: \overline{\mu^C_{z,n}}$	$\forall$ z, n	(20)

The correctness of the presented method relies on the uniqueness of the inner model, i.e., the spot-market. This is, however, not the case because variable production costs do not differ within a technology group and no unique dispatch rule exists for generators with the same variable costs at different locations. To circumvent this limitation, I introduce quadratically increasing generation costs for each technology at each location similar to price adder on capacity costs. While these adders only have little impact on the overall outcome, they guarantee uniqueness of the dispatch.

Summary. The inner problem hence describes the profit maximization of agents in a zonal electricity, which accounts for regulatory locational signals (from the outer problem). The inner problem is summarized as follows:

Maximize private profits (absent of network costs and including the locational instrument)

s.t. Zonal energy balance (15)

Price-elasticity of demand (3)

Production constraints (16-20)

2.3.3. *Solution strategy and problem reformulation*

Reformulation. The choice of the welfare maximizing locational signal is subject to the inner equilibrium defined in Subsection 2.3.2. Such a Stackelberg model can be reformulated into a single level by formulating the optimality (KKT) conditions of the lower-level problem and inserting these as constraints in the outer problem.[3] The reformulation leads to the Mathematical Programs with Equilibrium Constraints (MPEC) described by Equations 21-36.

$$\max_{c^{instr}_{z,n}, \overline{f_l}, f_{l,t}, R^{up}_{t,z,n}, R^{down}_{t,z,n}, G_{t,z,n}, P_{z,n}, d_{t,n}} \sum_{t,n}\left[p^{ref}_t \cdot d_{t,n} \cdot \left(1 - \frac{1}{\varepsilon} + \frac{d_{t,n}}{2\varepsilon \cdot d^{ref}_{t,n}}\right)\right] - \sum_{z,n}\left[P_{z,n} \cdot \left(c^{fix}_{z,n} + P_{z,n} \cdot m_s\right)\right] - \sum_{t,z,n} G_{t,z,n} \cdot c^{var}_{z,n} - \sum_l \overline{f}_l \cdot c^{grid}_l - \sum_{t,z,n}\left[R^{up}_{t,z,n} \cdot c^{var}_{z,n} + R^{down}_{t,z,n} \cdot \left(\lambda_t - c^{var}_{z,n}\right)\right] \quad (21)$$

s.t.

$\sum_z\left[G_{t,z,n} + R^{up}_{t,z,n} - R^{down}_{t,z,n}\right] - d_{t,n} = \sum_{l\ out} f_{l,t} + \sum_{l\ in} f_{l,t}$	$\forall$ t, l, n	(22)
$f_{i\to j,t} = B \cdot \left(\theta_{i,t} - \theta_{j,t}\right)$	$\forall$ t, (i, j) $\in$ N	(23)
$f_{i\to j,t} \leq \overline{f_{i\to j}}$	$\forall$ t, (i, j) $\in$ N	(23)
$\theta_{n^*,t} = 0$	$\forall$ t	(24)

[3] Mathematically, the inner problem is a quadratic minimization problem with linear constraints. Its Karush-Kuhn-Tucker (KKT) conditions are therefore both necessary and sufficient for optimality.

$$R_{t,z,n}^{down} \leq G_{t,z,n} \qquad \forall\, \mathrm{t,z,n} \qquad (25)$$

$$R_{t,z,n}^{up} \leq P_{z,n} \cdot \alpha_{t,z,n} - G_{t,z,n} \qquad \forall\, \mathrm{t,z,n} \qquad (26)$$

Complementarity constraints of the inner problem:

$$\frac{\partial \mathcal{L}}{\partial G_{t,z,n}} = c_{z,n}^{var} + \overline{\mu_{t,z,n}^{G}} - \underline{\mu_{t,z,n}^{G}} - \lambda_t = 0 \;\; \forall\, \mathrm{t,z,n} \qquad (27)$$

$$\frac{\partial \mathcal{L}}{\partial P_{z,n}} = c_{z,n}^{fix} + 2 \cdot m_s \cdot P_{z,n} + S_{z,n} - \sum_{t} \overline{\mu_{t,z,n}^{G}} \cdot \alpha_{t,z,n} + \overline{\mu_{z,n}^{C}} - \underline{\mu_{z,n}^{C}} = 0\; \forall\, \mathrm{z,n} \qquad (28)$$

$$\frac{\partial \mathcal{L}}{\partial d_{t,n}} = -p_t^{ref} \cdot \left(1 - \frac{1}{\varepsilon}\right) - \frac{p_t^{ref} \cdot d_{t,n}}{\varepsilon \cdot d_{t,n}^{ref}} - \underline{\mu_{t,n}^{D}} + \lambda_t = 0 \qquad \forall\, \mathrm{t,n} \qquad (29)$$

$$\sum_{\mathrm{n}} d_{t,n} - \sum_{\mathrm{z,n}} G_{t,z,n} = 0 \qquad \forall\, \mathrm{t} \qquad (30)$$

$$G_{t,z,n} - P_{z,n} \cdot \alpha_{t,z,n} \;\perp\; \overline{\mu_{t,z,n}^{G}} \qquad (31)$$

$$-G_{t,z,n} \;\perp\; \underline{\mu_{t,z,n}^{G}} \qquad (32)$$

$$P_{z,n} - \overline{P}_{z,n} \;\perp\; \overline{\mu_{z,n}^{C}} \qquad (33)$$

$$-P_{z,n} \;\perp\; \underline{\mu_{z,n}^{C}} \qquad (34)$$

$$-\,d_{t,n} \;\perp\; \underline{\mu_{t,n}^{D}} \qquad (35)$$

$$\lambda_t \text{ free} \qquad (36)$$

Simplification strategies. Mathematically, the resulting model is a nonlinear problem (NLP), which implies that most commercial solvers do not guarantee to find the global optimum of the problem. The nonlinearities arise from product of downward redispatch $R_{t,z,n}^{down}$ and electricity price λ_t, the complementarity constraints, and the quadratic terms in the objective function. Because NLPs are NP-hard, several techniques are proposed in literature to convert these models into mixed integer linear programs (MILP). Sometimes, bilinear terms can be linearized by exploiting the strong duality theorem, e.g. Zugno et al. (2013) and Garces et al. (2009). In the model presented here, this approach is not feasible because the product of $R_{t,z,n}^{down}$ (primal variable of the outer problem) and λ_t (dual variable of the inner problem) does not appear in any of the KKT conditions. An alternative is the McCormick Envelope, which allows the estimation of a lower bound of the objective function. This approach is also not applied here because I am aim at characterizing the optimum as a whole and not only the objective function.

Approach. To obtain a solution, I simplify the problem. First, I relax the non-linearity in the objective function $R_{t,z,n}^{down} \cdot \left(\lambda_t - c_{z,n}^{var}\right)$ by replacing the market clearing price λ_t by the exogenous parameter p_t^{ref} (the reference electricity price). Further, I linearize the complementarity constraints with the Fortuny-Amat (or "Big M") method presented by Fortuny-Amat and McCarl (1981). The resulting problem is a mixed integer quadratic problem (MIQP), which can be solved by the CPLEX solver. Subsequently, I use the results from the relaxed problem as starting point for the original (non-relaxed) problem with an NLP solver: I use the GAMS solver CONOPT3. The aim of this paper is to analyze structural characteristics of locational signals based on small-scale examples. I therefore refrain from further model reformulations and simplifications, which remains an interesting avenue for further research.

Uniqueness of the equilibrium. I close this section with a brief remark on the solutions found by the presented approach. The presented Stackelberg game has one single equilibrium if the lower level has a unique solution. The increasing variable and fixed costs of each technology at each location ensure such a unique solution.

3. Illustrative example

Setup. I apply the model to an illustrative power system to uncover fundamental properties of locational investment signals. The modeled system comprises of two regions interlinked by a transmission line. I call the regions North and South. The greenfield model allows investment in four stylized technologies, namely wind, solar, baseload, and a peaking technology. Investment and variable costs are presented in Table 2.

Table 2: Assumptions for variable and fixed costs

	c^{var}	c^{fix}
	€/MWh	€/ kW per a
Base	55	95
Peak	80	32
Onshore wind	-	85
Solar	-	50

Investment in transmission infrastructure is endogenous and assumes annualized investment costs of $180\text{k}\ ^{€}/_{MW}$ per annum. The availabilities of wind and solar power differ between the two locations and are loosely calibrated with data from northern and southern Germany. For simplicity, the two conventional technologies are assumed to be always available. The model covers 48 timesteps. To calculate the consumer welfare and the price-elastic electricity demand, a reference price of EUR 60 per MWh is assumed. As the availability factors, the reference load also varies over time. The price-elasticity of demand ε is -0.25, i.e., a 10% increase in the electricity price leads to a reduction of demand by 2.5%. Table X in Annex A lists reference demand level and availability factors for wind and solar for each modeled hour.

Three scenarios. To analyze the effect of well-designed investment signals, I compare three scenarios.

- The reference scenario is the market equilibrium arising in a *single market zone* in which dispatch and investment incentives are sub-optimal (Implementation in Annex A2.1).
- The *second-best equilibrium* represents the introduction of welfare-optimal locational investment signals in a zonal market: Investment incentives are optimal given the (foreseeable) sub-optimal dispatch incentives arising from a single bidding zone.
- The *first-best equilibrium*, which serves as a theoretical benchmark, is a scenario in which both dispatch and investment incentives are optimal. It is implemented as a nodal market with optimal network investment (Implementation in Annex A2.2). Under the absence of strategic behavior, perfect foresight of all agents, and optimal transmission investment, such a nodal market internalizes all network costs.

4. Results and discussion

Summary. The model results reveal significant welfare gains through an optimal placement of generators compared to the reference scenario of zonal market. This improvements account for nearly half of the improvements that would arise under optimal siting and optimal dispatch incentives. Further, I find that the best placement of generators, given the suboptimal dispatch resulting from a zonal market, differs from the first-based placement if the dispatch is optimal. In the following section, I discuss all findings in detail.

4.1. Cost and welfare analysis

Cost-shift. Table 3 provides an overview of the numerical results for the analyzed system. They highlight the rational of the locational signals: Through the internalization of network costs, generators are constructed at sites where generation costs are slightly higher than without locational signals. This increase in costs is overcompensated by a strong reduction in the network costs. As discussed in previous sections, the resulting distribution of generators is optimal given the suboptimal dispatch incentives.

Welfare analysis. The welfare analysis shows that the optimal siting of generators in a zonal market (second-best equilibrium) increases welfare by 5.5 % compared to the reference scenario. This is a significant part of the benefits of the first-best equilibrium where both dispatch and investment incentives are optimal. The benchmark of the first-best equilibrium corresponds to a nodal electricity market with optimal network investment. It features a welfare improvement of 8.0 % compared to the reference scenario. This example demonstrates that the introduction of locational investment signals can partly compensate the shortcomings of a zonal market compared to a nodal market. Nevertheless, even with locational signals, zonal markets lack adequate dispatch incentives and local incentives for demand flexibility

Consumer surplus. The evaluation of the consumer surplus shows that the introduction of locational signals leads to significantly higher generation costs comparted to the reference scenario. This can be explained by the fact that the locational signal internalizes the network costs and thereby increases electricity prices, which lowers the consumer surplus.

Table 3: Cost and welfare analysis

	Network cost	Generation cost	Gross consumer surplus	Social welfare
Single pricing zone (Reference)	80	308	1,000	614
Second-best equilibrium (locational investment signals)	8	324	990	648
First-best equilibrium (Benchmark)	4	328	998	668

4.2. Placement of generators

Placement in the reference scenario. Figure 3 shows the placement of generators for the three analyzed scenarios. The single market zone without locational signals results in a suboptimal placement of generators, which, in turn, leads to excessive line investment. In this reference scenario, the siting of renewable energy sources is purely driven by their market values, which mostly depend on the resource availability at the different sites. This leads to a strong concentration of these technologies; wind in the north and solar in the south. Conventional generators do not depend wind speeds and solar potential and are therefore indifferent for where to locate, reason why I introduced the linearly increasing capacity costs, which lead to an equal distribution between north and south.

Optimal placement. The welfare-optimal placement of generators depends on whether also dispatch incentives are optimal or not. In the first-best equilibrium, network constraints are already accounted for in the dispatch decisions. This is not the case in a zonal market (second-best equilibrium), where the lack of adequate dispatch incentives makes redispatch measures necessary, which leads to additional costs. Intuitively, these additional costs imply that it is better to locate generation and demand closer together in zonal markets as compared to in the nodal benchmark, visible in Figure 3. The figure also shows that also the generation mix differs between the first-best and the second-best equilibrium. In a zonal electricity market, base-load generators lead to the highest network costs and are therefore penalized, which is also shown in the next section.

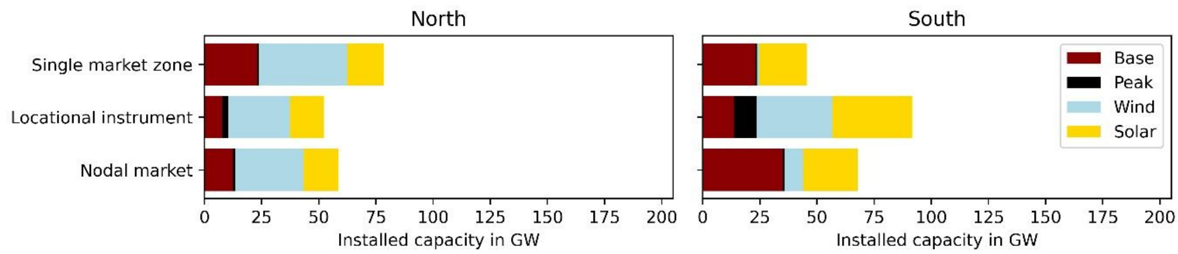

Figure 3: Capacity mix and regional distribution by technology

Research implications. This finding implies that the optimal distribution of generators in a zonal market is not possible with a zonal model as it is often done. In addition, locational signals for both generators and consumers that are calculated based on nodal prices, are not optimal. Because the siting of generation and demand is further apart in nodal markets, locational signals derived from nodal prices structurally underestimate the optimal locational signal. Instead, addressing these research questions requires a model such as presented in this paper.

4.3. Analysis: Instrument level

Cost-optimal locational signal. One outcome of the model is the locational signal that maximizes welfare in the power system. Striking are the high observed locational signals for baseload generators which reflects fact that their inflexible dispatch in a zonal electricity market increases network costs significantly. This is, however, also partly a model artifact: To guarantee unique solutions in the inner model, I assume that generation is distributed equally between sites to the degree that capacity is available. Since the dispatch can only be affected through the construction of less baseload plants, the signal is so strong that it reduces the overall share of baseload in the system to lower network costs.[4]

	North		South		C_{fix}
	Signal (€/kW)	Share of C_{fix} (%)	Signal (€/kW)	Share of C_{fix} (%)	(€/kW)
base	206 €	40%	187 €	36%	521 €
peak	20 €	11%	21 €	12%	175 €
solar	7 €	3%	12 €	5%	274 €
wind	181 €	39%	-17 €	-4%	466 €

Further findings. Three more general findings are worth discussing. First, the results indicate that the optimal locational signal is location-specific and technology-specific. Due to the different generation profiles, some technologies result in higher network costs than others, which is reflected in the diverging level of charges. This implies that locational signals should ideally differentiate by technology, which is not the case in most implemented instruments (Eicke, Khanna, and Hirth 2020). Second, the presented methodology estimates long-run equilibria of the power system, with continuous transmission investment. The signal leading to this equilibrium may differ significantly from the short-run signals arising from nodal prices which reflect the current state of the transmission infrastructure. Third, locational signals are only calculated for generators in this study. Similarly, locational signals could be calculated for specific consumers, e.g., electrolyzers. Due to the lack of a temporal and locational price signal in zonal markets, I expect energy intensive consumers to be ideally located closer to producers compared to a nodal market.

Relation to previous literature. These findings are in line with previous literature that suggests are particularly high impact of RES on network costs due to their intermittency and remote locations (Joos and Staffell 2018; Hitaj and Löschel 2019). Similarly, Costa-Campi et al. (2020) argue that technology type affects how much the location of generators impacts flows in the transmission network. For the case of Spain, the authors emphasize that wind generators and imports are the least efficiently located with respect to the main consumption areas, while combined cycle plants are most efficiently located.

4.4. Discussion

Where to apply locational signals? Locational investment signals are worth considering in all power systems where the network costs can be lowered through a relocation of generators or consumers. This contribution analyses the effect of locational signals in power systems with a zonal market design, but even in nodal markets, locational investment signals can be of relevance. Because nodal market prices also do not lead to optimal investment incentives[5] in practice, such markets would also benefit from regulatory determined long-term locational signals.

[4] More work needs to be done to find a satisfactory solution to this problem.

[5] Because the assumption of optimal transmission investment never holds in practice (Pérez-Arriaga et al. 1995), locational signals arising from nodal markets only reflect the short-term but not the long-term value differences between locations.

Price vs. quantity signals. In this contribution, I focused on price signals because they allowed the implementation in a bilevel model, where the regulator only determines the price signal, but investment decisions remain with independent agents. In practice, a locational steering of generation capacity can also take place in the form of quantity-based mechanisms such as land restrictions or land-use planning.

Theoretical potential. The analysis highlights the large cost-saving potential of locational investment signals in zonal power markets. However, these signals are determined by the regulator and do not arise from the market. Consequently, this regulatory intervention is sensitive to input data and prone to political influence. The model results are thus an upper bound of the potential of locational instruments, which remains unachievable given the uncertainty in future investments, commodity prices, changing weather patterns and future technology developments. Despite all these limitations, it is unlikely that the regulatory interventions do not result in an improvement compared to highly suboptimal setting of a zonal power market without additional investment signals.

Implementation challenges. Practical challenges arise when introducing locational signals in interconnected power systems. Cost-reflective locational instruments impose additional costs on generators, which imposes market-distortions at the borders with systems that do not apply locational instruments. More research is required on these border-effects. This also includes the political question to which extent generators can be moved to neighboring countries if that reduces overall system costs. These questions do not arise in isolated countries such as island-systems, where the introduction of locational signals is thus easier.

Value of the model. In the exemplary power system, the cost-saving potential of optimal investment incentives is significant even when the dispatch remains suboptimal. If adequately designed, locational signals lower the overall system costs through a better allocation of power generation. These findings are, however, specific to each power system. Consider for example the case of a system in which the availability of renewables and the electricity demand does not differ between regions. In this illustrative example, it is easy to understand that a reallocation of generators cannot lower the overall system costs. The fact that the added benefit of locational instruments strongly depends on the underlying system highlights the value of the presented modelling approach.

Limitations. The presented model allows the determination of the long-term equilibrium resulting from the optimal siting of generators resulting from the internalization of network costs. Such an approach reveals the possible benefit of locational signals. The approach is, however, far too complex to be used for the practical implementation of locational signals and, besides, estimates the optimal signal in the equilibrium only. In practice, such locational signals should base on an approximation of the network costs resulting from new generators.

5. Summary and conclusions

Novel methodology. The main contribution of this paper is its novel methodological approach, which allows the estimation of cost-optimal, location-specific investment incentives in a zonal electricity market. By modelling the interplay between a welfare maximizing regulator and profit maximizing private firms, I determine the optimal placement of generators given the suboptimal dispatch resulting from a uniform pricing zone.

Cost-saving potential. In zonal markets, locational signals can internalize the network costs resulting from the siting decisions of generators and consumers. The results for an illustrative power market model reveal that the cost-saving potential of locational instruments in zonal power markets can be significant: Through a relocation of generators, locational investment signals lower the overall system costs. However, because investment signals are time-invariant in practice, they do not provide the same quality of dispatch incentives as nodal markets. The resulting placement of generators therefore remains a second-best solution in terms of locational incentives.

Charges should be technology-specific. The exemplary power market model indicates that the welfare-optimal locational signal is not only location-specific but also technology-specific. Due to the different generation profiles, some technologies result in much higher network costs than others, which is reflected in the divergent levels of charges. This implies that locational signals should differentiate between technologies, which is, however, not the case in most real-world instruments.

Optimal distribution of generators. The results also show that the optimal geographical distribution and the optimal technological mix of generators depends on the electricity market design. Because the dispatch signals in a nodal market reflect network constraints, these signals lower the cost of network expansion. In zonal markets, by contrast, the lack of adequate dispatch incentives introduces higher costs for managing the network. The optimal siting of generation in zonal markets is therefore closer to demand centers compared to nodal power systems.

Outlook. The presented methodology of a reformulated bilevel model cannot be applied to real-world power systems in its current state because of its computationally demanding properties. An avenue for further research could be to take further simplifying assumptions that facilitate the model's mathematical solution and then apply it to real world systems. Additionally, the model could be applied to a nodal electricity market with imperfect network investment.

Acknowledgements

I would like to thank Raffaele Sgarlato, Andreas Fleischhacker, Lukas Lang, Daniel Huppmann, and Li Bai for fruitful discussions, valuable comments, and feedback. Previous versions of this paper were presented at the first international IAEE conference 2021, the IEWT 2021, and at the Energy Research Seminar at the Hertie School. I thank for valuable feedback from all participants. Finally, I highly acknowledge Jalal Kazempour's inspiring lecture videos on "Advanced Optimization and Game Theory for Energy Systems".

References

Borenstein, Severin. 2012. "The Private and Public Economics of Renewable Electricity Generation." *Journal of Economic Perspectives* 26 (1): 67–92. https://doi.org/10.1257/jep.26.1.67.

Brown, David P., Jay Zarnikau, and Chi-Keung Woo. 2020. "Does Locational Marginal Pricing Impact Generation Investment Location Decisions? An Analysis of Texas's Wholesale Electricity Market." *Journal of Regulatory Economics* 58 (2–3): 99–140. https://doi.org/10.1007/s11149-020-09413-0.

Costa-Campi, Maria Teresa, Daniel Davi-Arderius, and Elisa Trujillo-Baute. 2020. "Locational Impact and Network Costs of Energy Transition: Introducing Geographical Price Signals for New Renewable Capacity." *Energy Policy* 142 (July): 111469. https://doi.org/10.1016/j.enpol.2020.111469.

Eicke, Anselm, Tarun Khanna, and Lion Hirth. 2020. "Locational Investment Signals: How to Steer the Siting of New Generation Capacity in Power Systems?" *The Energy Journal* 41 (01). https://doi.org/10.5547/01956574.41.6.aeic.

Fortuny-Amat, José, and Bruce McCarl. 1981. "A Representation and Economic Interpretation of a Two-Level Programming Problem." *Journal of the Operational Research Society* 32 (9): 783–92. https://doi.org/10.1057/jors.1981.156.

Garces, L.P., A.J. Conejo, R. Garcia-Bertrand, and R. Romero. 2009. "A Bilevel Approach to Transmission Expansion Planning Within a Market Environment." *IEEE Transactions on Power Systems* 24 (3): 1513–22. https://doi.org/10.1109/TPWRS.2009.2021230.

Grimm, Veronika, Alexander Martin, Martin Schmidt, Martin Weibelzahl, and Gregor Zöttl. 2016. "Transmission and Generation Investment in Electricity Markets: The Effects of Market Splitting and Network Fee Regimes." *European Journal of Operational Research* 254 (2): 493–509. https://doi.org/10.1016/j.ejor.2016.03.044.

Grimm, Veronika, Bastian Rückel, Christian Sölch, and Gregor Zöttl. 2019. "Regionally Differentiated Network Fees to Affect Incentives for Generation Investment." *Energy* 177 (June): 487–502. https://doi.org/10.1016/j.energy.2019.04.035.

Hitaj, Claudia, and Andreas Löschel. 2019. "The Impact of a Feed-in Tariff on Wind Power Development in Germany." *Resource and Energy Economics* 57 (August): 18–35. https://doi.org/10.1016/j.reseneeco.2018.12.001.

Joos, Michael, and Iain Staffell. 2018. "Short-Term Integration Costs of Variable Renewable Energy: Wind Curtailment and Balancing in Britain and Germany." *Renewable and Sustainable Energy Reviews* 86 (April): 45–65. https://doi.org/10.1016/j.rser.2018.01.009.

Neuhoff, Karsten, Julian Barquin, Janusz W. Bialek, Rodney Boyd, Chris J. Dent, Francisco Echavarren, Thilo Grau, et al. 2013. "Renewable Electric Energy Integration: Quantifying the Value of Design of Markets for International Transmission Capacity." *Energy Economics* 40 (November): 760–72. https://doi.org/10.1016/j.eneco.2013.09.004.

Pérez-Arriaga, Ignacio José, Francisco Javier Rubio Odériz, Juan Felipe Puerta Guitérrez, José Arcélus Ogando, and Javier Martin. 1995. "Marginal Pricing of Transmission Services. An Analysis of Cost Recovery" IEEE Transactions on Power Systems (1): 65–72. https://doi.org/10.1109/59.373981.

Samuelson, Paul A. 1952. "Spatial Price Equilibrium and Linear Programming." *The American Economic Review* 42 (3): 283–303.

Scheidt, Frederik vom, Jingyi Qu, Philipp Staudt, Dharik S. Mallapragada, and Christof Weinhardt. 2021. "Integrating Hydrogen in Single-Price Electricity Systems: The Effects of Spatial Economic Signals." *ArXiv:2105.00130 [Econ, q-Fin]*, April. http://arxiv.org/abs/2105.00130.

Schmidt, Lukas, and Jonas Zinke. 2020. "One Price Fits All? Wind Power Expansion under Uniform and Nodal Pricing in Germany." *EWI Working Paper* No 20/06.

Zugno, Marco, Juan Miguel Morales, Pierre Pinson, and Henrik Madsen. 2013. "A Bilevel Model for Electricity Retailers' Participation in a Demand Response Market Environment." *Energy Economics* 36 (March): 182–97. https://doi.org/10.1016/j.eneco.2012.12.010.

Annex A1: Lagrange function of the inner problem

$$\mathcal{L}\,(G_{t,z,n}, P_{z,n}, d_{t,n}, \overline{\mu^{G}_{t,z,n}}, \underline{\mu^{G}_{t,z,n}}, \overline{\mu^{C}_{z,n}}, \underline{\mu^{C}_{z,n}}, \lambda_t)$$

$$\begin{aligned}
&= \sum_{z,n}\left[P_{z,n}\cdot\left(c^{fix}_{z,n} + P_{z,n}\cdot m_s + S_{z,n}\right)\right] + \sum_{t,z,n}\left(G_{t,z,n}\cdot c^{var}_{z,n}\right) \\
&- \sum_{t,n} p^{ref}_t \cdot d_{t,n}\cdot\left(1-\frac{1}{\varepsilon}+\frac{d_{t,n}}{2\varepsilon\cdot d^{ref}_{t,n}}\right) + \sum_{\boldsymbol{t,z,n}} \overline{\mu^{G}_{t,z,n}}\cdot\left[G_{t,z,n}-\alpha_{t,z,n}\cdot P_{z,n}\right] \\
&- \sum_{t,z,n}\underline{\mu^{G}_{t,z,n}}\cdot G_{t,z,n} + \sum_{\boldsymbol{z,n}}\overline{\mu^{C}_{z,n}}\cdot\left[P_{z,n}-\bar{P}_{z,n}\right] - \sum_{z,n}\underline{\mu^{C}_{z,n}}\cdot P_{z,n} - \sum_{t,n}\underline{\mu^{D}_{t,n}}\cdot d_{t,n} \\
&+ \sum_{t}\lambda_t\cdot\left[\sum_{\mathrm{n}} d_{t,n} - \sum_{\mathrm{z,n}} G_{t,z,n}\right]
\end{aligned}$$

Annex A2: Reference models

A2.1 Zonal power system without locational signals (Reference)

The reference scenario of a zonal power system can be modeled by setting the locational signals to zero. This also allows to simplify the model into two subsequent problems: first, the market clearing is calculated and with the outcome the cost-optimal network is determined.

A2.2 Nodal power system (Benchmark)

As benchmark, I consider a nodal-pricing model in which all investment, dispatch, and demand decisions are taken simultaneously by a fictitious integrated generation and transmission company that maximizes social welfare. The social welfare function – the difference between gross consumer surplus and generation and network costs is the same as in the outer model.

Annex A3: Inverse demand function

For the analysis, I assume a linear inverse demand function with a negative slope $(b < 0)$

$$p(d) = a + b\cdot d,$$

which is equivalent to the following demand function

$$d(p) = -\tfrac{a}{b} + \tfrac{1}{b}\cdot p.$$

Using the definition of demand elasticity ε

$$\varepsilon = \tfrac{\delta d}{\delta p}\cdot\tfrac{p}{d} = \tfrac{1}{b}\cdot\tfrac{p}{d},$$

I obtain slope b and intercept a

$$b = \frac{p^{ref}}{d^{ref}}\cdot\frac{1}{\varepsilon}$$

$$a = p^{ref} - b \cdot d^{ref}.$$

and finally the following demand function

$$p(d) = p^{ref} \cdot \left[1 - \frac{1}{\varepsilon} + \frac{d}{\varepsilon \cdot d^{ref}}\right].$$